Spring 2012

EE 330

ENGINEERING ELECTROMAGNETICS

HW 13: *Due Friday 27 April* ***(LAST ONE!!!)***
8.36, 9.10, 9.17, 9.19, 9.20, 9.29, 9.30, 9.36, 9.38

Problem 8.36 A 50-MHz right-hand circularly polarized plane wave with an electric field modulus of 30 V/m is normally incident in air upon a dielectric medium with $\varepsilon_r = 9$ and occupying the region defined by $z \geq 0$.

(a) Write an expression for the electric field phasor of the incident wave, given that the field is a positive maximum at $z = 0$ and $t = 0$.

(b) Calculate the reflection and transmission coefficients.

(c) Write expressions for the electric field phasors of the reflected wave, the transmitted wave, and the total field in the region $z \leq 0$.

(d) Determine the percentages of the incident average power reflected by the boundary and transmitted into the second medium.

Solution:

(a)

$$\begin{aligned}
k_1 &= \frac{\omega}{c} = \frac{2\pi \times 50 \times 10^6}{3 \times 10^8} = \frac{\pi}{3} \text{ rad/m}, \\
k_2 &= \frac{\omega}{c}\sqrt{\varepsilon_{r_2}} = \frac{\pi}{3}\sqrt{9} = \pi \text{ rad/m}.
\end{aligned}$$

From (7.57), RHC wave traveling in $+z$ direction:

$$\begin{aligned}
\widetilde{\mathbf{E}}^{\mathrm{i}} &= a_0(\hat{\mathbf{x}} + \hat{\mathbf{y}}\, e^{-j\pi/2})e^{-jk_1 z} = a_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{-jk_1 z} \\
\mathbf{E}^{\mathrm{i}}(z,t) &= \mathfrak{Re}\left[\widetilde{\mathbf{E}}^{\mathrm{i}} e^{j\omega t}\right] \\
&= \mathfrak{Re}\left[a_0(\hat{\mathbf{x}}\, e^{j(\omega t - k_1 z)} + \hat{\mathbf{y}}\, e^{j(\omega t - k_1 z - \pi/2)})\right] \\
&= \hat{\mathbf{x}}\, a_0 \cos(\omega t - k_1 z) + \hat{\mathbf{y}}\, a_0 \cos(\omega t - k_1 z - \pi/2) \\
&= \hat{\mathbf{x}}\, a_0 \cos(\omega t - k_1 z) + \hat{\mathbf{y}}\, a_0 \sin(\omega t - k_1 z) \\
|\mathbf{E}^{\mathrm{i}}| &= \left[a_0^2 \cos^2(\omega t - k_1 z) + a_0^2 \sin^2(\omega t - k_1 z)\right]^{1/2} = a_0 = 30 \text{ V/m}.
\end{aligned}$$

Hence,

$$\widetilde{\mathbf{E}}^{\mathrm{i}} = 30(x_0 - jy_0)e^{-j\pi z/3} \quad \text{(V/m)}.$$

(b)

$$\eta_1 = \eta_0 = 120\pi \quad (\Omega), \qquad \eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r_2}}} = \frac{120\pi}{\sqrt{9}} = 40\pi \quad (\Omega).$$

$$\begin{aligned}
\Gamma &= \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{40\pi - 120\pi}{40\pi + 120\pi} = -0.5 \\
\tau &= 1 + \Gamma = 1 - 0.5 = 0.5.
\end{aligned}$$

(c)

$$\begin{aligned}
\widetilde{\mathbf{E}}^{\rm r} &= \Gamma a_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{jk_1z} \\
&= -0.5 \times 30(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{jk_1z} \\
&= -15(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{j\pi z/3} \quad \text{(V/m)}. \\
\widetilde{\mathbf{E}}^{\rm t} &= \tau a_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{-jk_2z} \\
&= 15(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{-j\pi z} \quad \text{(V/m)}. \\
\widetilde{\mathbf{E}}_1 &= \widetilde{\mathbf{E}}^{\rm i} + \widetilde{\mathbf{E}}^{\rm r} \\
&= 30(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{-j\pi z/3} - 15(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{j\pi z/3} \\
&= 15(\hat{\mathbf{x}} - j\hat{\mathbf{y}})[2e^{-j\pi z/3} - e^{j\pi z/3}] \quad \text{(V/m)}.
\end{aligned}$$

(d)

$$\% \text{ of reflected power} = 100 \times |\Gamma|^2 = 100 \times (0.5)^2 = 25\%$$

$$\% \text{ of transmitted power} = 100|\tau|^2 \frac{\eta_1}{\eta_2} = 100 \times (0.5)^2 \times \frac{120\pi}{40\pi} = 75\%.$$

Problem 9.10 The normalized radiation intensity of a certain antenna is given by

$$F(\theta) = \exp(-20\theta^2) \qquad \text{for } 0 \le \theta \le \pi$$

where θ is in radians. Determine:

(a) The half-power beamwidth.

(b) The pattern solid angle.

(c) The antenna directivity.

Solution:

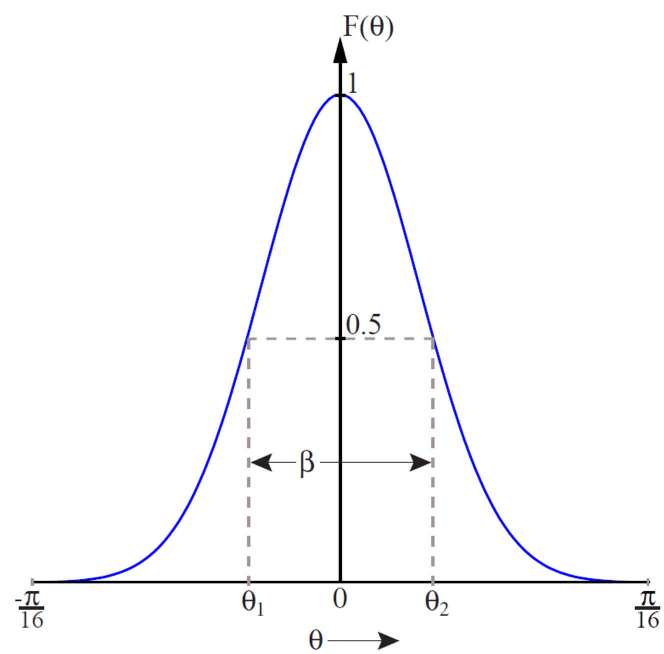

Figure P9.10: $F(\theta)$ versus θ.

(a) Since $F(\theta)$ is independent of ϕ, the beam is symmetrical about $z = 0$. Upon setting $F(\theta) = 0.5$, we have

$$\begin{aligned}
F(\theta) &= \exp(-20\theta^2) = 0.5, \\
\ln[\exp(-20\theta^2)] &= \ln(0.5), \\
20\theta^2 &= -0.693,
\end{aligned}$$

$$\theta = \pm\left(\frac{0.693}{20}\right)^{1/2} = \pm 0.186 \text{ radians.}$$

Hence, $\beta = 2 \times 0.186 = 0.372$ radians $= 21.31°$.

(b) By Eq. (9.21),

$$\begin{aligned}
\Omega_p &= \iint_{4\pi} F(\theta)\sin\theta\, d\theta\, d\phi \\
&= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \exp(-20\theta^2)\sin\theta\, d\theta\, d\phi \\
&= 2\pi \int_0^{\pi} \exp(-20\theta^2)\sin\theta\, d\theta.
\end{aligned}$$

Numerical evaluation yields

$$\Omega_p = 0.156 \text{ sr.}$$

(c)

$$D = \frac{4\pi}{\Omega_p} = \frac{4\pi}{0.156} = 80.55.$$

Problem 9.17 Repeat parts (a)–(c) of Problem 9.15 for a dipole of length $l = \lambda$.

Solution: For $l = \lambda$, Eq. (9.56) becomes

$$S(\theta) = \frac{15I_0^2}{\pi R^2}\left[\frac{\cos(\pi\cos\theta) - \cos(\pi)}{\sin\theta}\right]^2 = \frac{15I_0^2}{\pi R^2}\left[\frac{\cos(\pi\cos\theta) + 1}{\sin\theta}\right]^2.$$

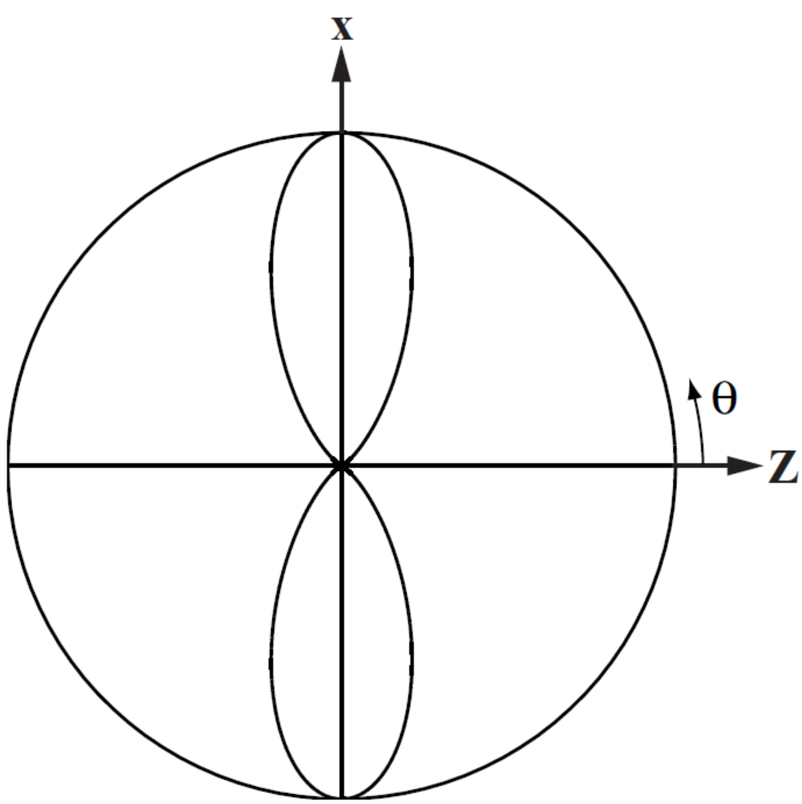

Figure P9.17: Radiation pattern of dipole of length $l = \lambda$.

Solving for the directions of maximum radiation numerically yields

$$\theta_{\max_1} = 90°, \qquad \theta_{\max_2} = 270°.$$

(b) From the numerical results, it was found that $S(\theta) = 15I_0^2/(\pi R^2)(4)$ at $\theta_{\max}$. Thus,

$$S_{\max} = \frac{60I_0^2}{\pi R^2}.$$

(c) The normalized radiation pattern is given by Eq. (9.13), as

$$F(\theta) = \frac{S(\theta)}{S_{\max}}.$$

Using the expression for $S(\theta)$ from part (a) with the value of $S_{\max}$ found in part (b),

$$F(\theta) = \frac{1}{4}\left[\frac{\cos(\pi\cos\theta)+1}{\sin\theta}\right]^2.$$

The normalized radiation pattern is shown in Fig. P9.17.

Problem 9.19 Determine the effective area of a half-wave dipole antenna at 100 MHz, and compare it with its physical cross-section if the wire diameter is 2 cm.

Solution: At $f = 100$ MHz, $\lambda = c/f = (3\times10^8 \text{ m/s})/(100\times10^6 \text{ Hz}) = 3$ m. From Eq. (9.47), a half wave dipole has a directivity of $D = 1.64$. From Eq. (9.64), $A_e = \lambda^2 D/4\pi = (3 \text{ m})^2 \times 1.64/4\pi = 1.17 \text{ m}^2$.

The physical cross section is: $A_p = ld = \frac{1}{2}\lambda d = \frac{1}{2}(3 \text{ m})(2\times10^{-2} \text{ m}) = 0.03 \text{ m}^2$. Hence, $A_e/A_p = 39$.

Problem 9.20 A 3-GHz line-of-sight microwave communication link consists of two lossless parabolic dish antennas, each 1 m in diameter. If the receive antenna requires 10 nW of receive power for good reception and the distance between the antennas is 40 km, how much power should be transmitted?

Solution: At $f = 3$ GHz, $\lambda = c/f = (3\times10^8 \text{ m/s})/(3\times10^9 \text{ Hz}) = 0.10$ m. Solving the Friis transmission formula (Eq. (9.75)) for the transmitted power:

$$\begin{aligned} P_t &= P_{rec}\frac{\lambda^2 R^2}{\xi_t \xi_r A_t A_r} \\ &= 10^{-8}\frac{(0.100 \text{ m})^2 (40\times10^3 \text{ m})^2}{1\times1\times(\frac{\pi}{4}(1 \text{ m})^2)(\frac{\pi}{4}(1 \text{ m})^2)} = 25.9\times10^{-2} \text{ W} = 259 \text{ mW}. \end{aligned}$$

Problem 9.29 An antenna with a circular aperture has a circular beam with a beamwidth of 3° at 20 GHz.

(a) What is the antenna directivity in dB?

(b) If the antenna area is doubled, what will be the new directivity and new beamwidth?

(c) If the aperture is kept the same as in (a), but the frequency is doubled to 40 GHz, what will the directivity and beamwidth become then?

Solution:

(a) From Eq. (9.96),

$$D \simeq \frac{4\pi}{\beta^2} = \frac{4\pi}{(3^\circ \times \pi/180^\circ)^2} = 4.59 \times 10^3 = 36.6 \text{ dB}.$$

(b) If area is doubled, it means the diameter is increased by $\sqrt{2}$, and therefore the beamwidth decreases by $\sqrt{2}$ to

$$\beta = \frac{3^\circ}{\sqrt{2}} = 2.2^\circ.$$

The directivity increases by a factor of 2, or 3 dB, to $D = 36.6 + 3 = 39.6$ dB.

(c) If f is doubled, λ becomes half as long, which means that the diameter to wavelength ratio is twice as large. Consequently, the beamwidth is half as wide:

$$\beta = \frac{3^\circ}{2} = 1.5^\circ,$$

and D is four times as large, or 6 dB greater, $D = 36.6 + 6 = 42.6$ dB.

Problem 9.30 A 94-GHz automobile collision-avoidance radar uses a rectangular-aperture antenna placed above the car's bumper. If the antenna is 1 m in length and 10 cm in height, determine the following:

(a) Its elevation and azimuth beamwidths.

(b) The horizontal extent of the beam at a distance of 300 m.

Solution:

(a) At 94 GHz, $\lambda = 3 \times 10^8/(94 \times 10^9) = 3.2$ mm. The elevation beamwidth is $\beta_e = \lambda/0.1 \text{ m} = 3.2 \times 10^{-2} \text{ rad} = 1.8^\circ$. The azimuth beamwidth is $\beta_a = \lambda/1 \text{ m} = 3.2 \times 10^{-3} \text{ rad} = 0.18^\circ$.

(b) At a distance of 300 m, the horizontal extent of the beam is

$$\Delta y = \beta_a R = 3.2 \times 10^{-3} \times 300 = 0.96 \text{ m}.$$

Problem 9.36 Find and plot the normalized array factor and determine the half-power beamwidth for a five-element linear array excited with equal phase and a uniform amplitude distribution. The interelement spacing is $3\lambda/4$.

Solution: Using Eq. (9.121),

$$F_{\rm an}(\theta) = \frac{\sin^2[(N\pi d/\lambda)\cos\theta]}{N^2\sin^2[(\pi d/\lambda)\cos\theta]} = \frac{\sin^2[(15\pi/4)\cos\theta]}{25\sin^2[(3\pi/4)\cos\theta]}$$

and this pattern is shown in Fig. P9.36. The peak values of the pattern occur at $\theta = \pm 90^\circ$. From numerical values of the pattern, the angles at which $F_{\rm an}(\theta) = 0.5$ are approximately 6.75°on either side of the peaks. Hence, $\beta \simeq 13.5^\circ$.

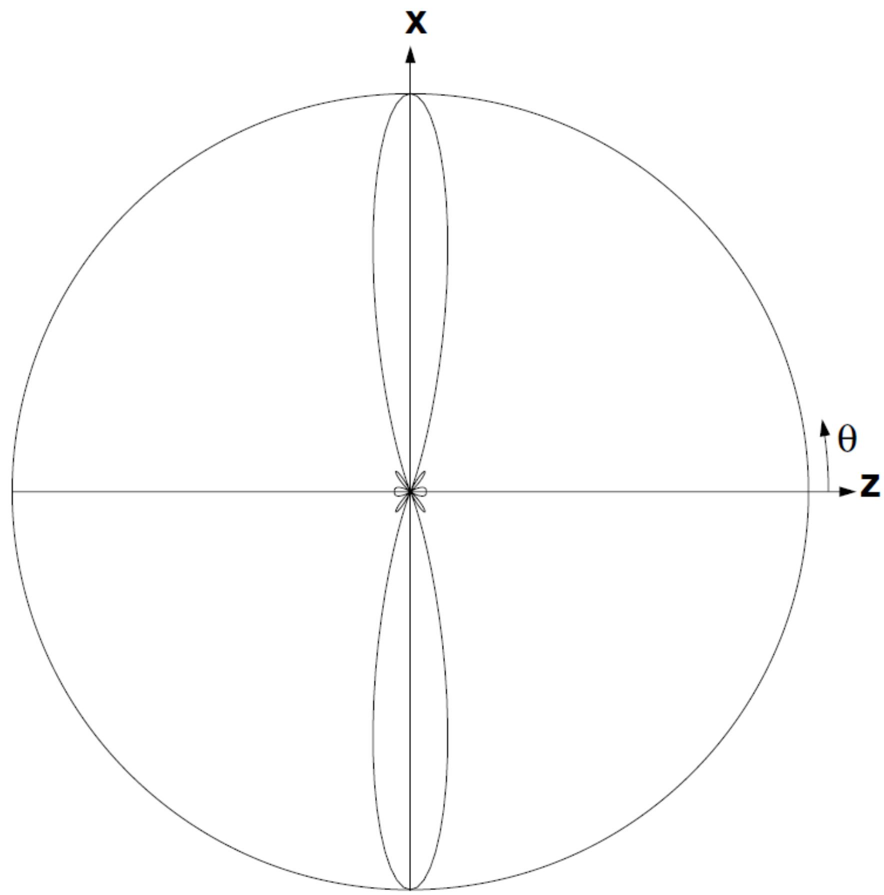

Figure P9.36: Normalized array pattern of a 5-element array with uniform amplitude distribution in Problem 9.36.

Problem 9.38 A three-element linear array of isotropic sources aligned along the z-axis has an interelement spacing of $\lambda/4$ (Fig. P9.38). The amplitude excitation of the center element is twice that of the bottom and top elements, and the phases are $-\pi/2$ for the bottom element and $\pi/2$ for the top element, relative to that of the center element. Determine the array factor and plot it in the elevation plane.

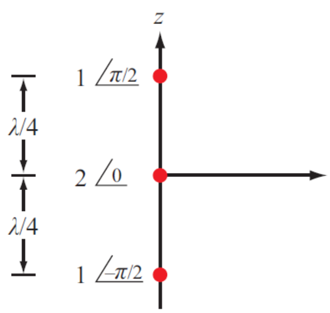

Figure P9.38: Three-element array of Problem 9.38.

Solution: From Eq. (9.110),

$$F_a(\theta) = \left|\sum_{i=0}^{2} a_i e^{j\psi_i} e^{jikd\cos\theta}\right|^2$$

$$= \left|a_0 e^{j\psi_0} + a_1 e^{j\psi_1} e^{jkd\cos\theta} + a_2 e^{j\psi_2} e^{j2kd\cos\theta}\right|^2$$

$$= \left|e^{j(\psi_1-\pi/2)} + 2e^{j\psi_1} e^{j(2\pi/\lambda)(\lambda/4)\cos\theta} + e^{j(\psi_1+\pi/2)} e^{j2(2\pi/\lambda)(\lambda/4)\cos\theta}\right|^2$$

$$= \left|e^{j\psi_1} e^{j(\pi/2)\cos\theta}\right|^2 \left|e^{-j\pi/2} e^{-j(\pi/2)\cos\theta} + 2 + e^{j\pi/2} e^{j(\pi/2)\cos\theta}\right|^2$$

$$= 4\left(1+\cos\left(\tfrac{1}{2}\pi(1+\cos\theta)\right)\right)^2,$$

$$F_{an}(\theta) = \tfrac{1}{4}\left(1+\cos\left(\tfrac{1}{2}\pi(1+\cos\theta)\right)\right)^2.$$

This normalized array factor is shown in Fig. 9.38(b).

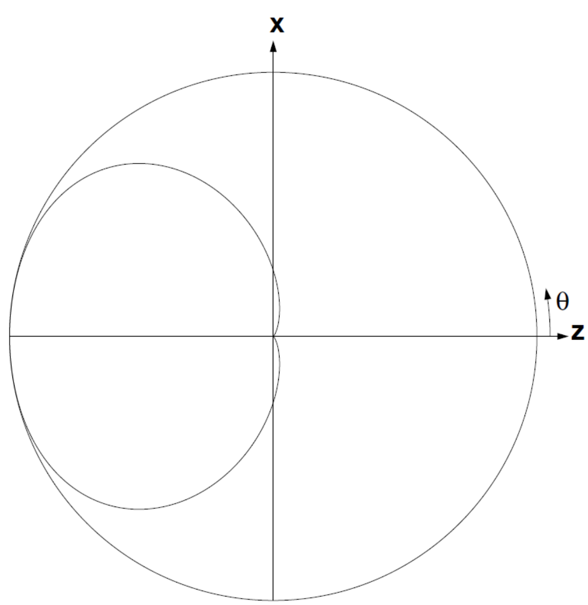

Figure P9.38: (b) Normalized array pattern of the 3-element array of Problem 9.38.